Paper ~~or~~ and digital: a study on combinatorics in preschool class

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In a design research study conducted in Swedish preschool class, children were asked: In how many ways can three toy bears sit in a sofa? In the initial design cycle the children who used pictographic representations seemed to make more systematic solutions with less duplications than children who used iconic representations. A later design cycle involved a digital application that was designed to provide an opportunity to work with pictographic representations on a semi-concrete level. In this paper, we make a comparison of paper and pencil documentation made by children who have used, respective not have used; the digital application. The results indicate that working with the digital application resulted in more systematic paper and pencil documentation with fewer duplications.

Keywords: Combinatorics, systematization, children's documentation, semi-concrete, digital application

Introduction

Studies have shown that appropriately designed and implemented activities enable young children to develop mathematical competencies that were earlier considered only attainable by older children (English & Mulligan, 2013). The use of multiple representations are considered to enhance young children's understanding of mathematics. Historically, mathematics education in the early years mainly involved numeracy and counting and this has therefore been the main area of research (Clements & Sarama, 2009). As part of this most previous studies on young children's representations have been connected to quantity with few studies on young children's use of representations when solving tasks within other mathematical areas (Hughes, 1986; Heddens; 1986).

In a preceding study we noticed that children who used an iconic representation when working on a combinatorics task made made more duplicate combinations than those using pictographic representations (Palmér & van Bommel, 2016). This was quite surprising as iconic representations are considered to be connected to a higher level of abstract thinking than pictographic representations (Hughes, 1986; Heddens, 1986). We also noticed that children's spontaneous use of representation lacked in systematisation. Therefore, the focus of the study presented in this paper became; *if* and *how* the use of an appropriately designed digital application influences the systematization and representation young children spontaneously use when they are working on a combinatorial task. The question asked to the children (age 6) was: In how many ways can three toy bears sit in a sofa: an enumerative combinatorial task where the children were supposed to count the permutations for n=3.

The paper is organised as follows: It starts with a presentation of the study's theoretical foundation. After that, the study itself and the two design cycles and their results are presented. Finally, some implications for further research are given.

Theoretical foundation

Representations have been used to describe children's work as well as mathematical objects per se. Hughes (1986) distinguished between idiosyncratic, pictographic, iconic and symbolic representations. Idiosyncratic representations are irregular and not related to the number of objects represented. Pictographic representations are pictures of the represented item. Iconic representations are based on one mark for each item. Symbolic representations are the standard forms like numerals or equal signs. Heddens (1986) focused instead on the connection between the concrete and abstract when analysing children's representations. He defined two levels, semi-concrete and semi-abstract, to describe representations used in between the concrete (objects) and the abstract (symbolic). At the semi-concrete level, pictures of real items, as a representation of the real situation, were considered. The semi-abstract level concerned a symbolic representation of the concrete items, with a constraint that the symbols would not look like the objects they represented. Thus, what Hughes (1986) named pictographic representations were semi-concrete in the wordings of Heddens (1986) whereas iconic representations were semi-abstract.

Listing items systematically has been shown to be difficult for young children when solving combinatorial tasks (English, 2005). Systematization refers to how the children organize their documentations when solving the task. A variety of graphic representations can be used when solving combinatorics task (for example lists, diagrams, sketches and tables) and these can be made systematic or not. English (1996) identified three stages of systematization when young children solve combinatorial tasks: the random stage, the transitional stage and the odometer stage. At the random stage children use trial-and-error and constant checking becomes important to succeed with a task. At the transition stage children start to adopt patterns in their documentations but the pattern is not kept all through the task, instead the children often revert to the trial-and error approach. At the odometer stage the children use an organized structure for the selection of combinations where one item is held constant while the others are varied systematically.

When analysing children's documentation when solving the combinatorial tasks in our study, we use English's (1996) notions *trial and error*, *transition* and *odometer* combined with Hughes' (1986) notions *pictographic* and *iconic* representations.

The study

The study took place in Swedish preschool class. In Sweden, the compulsory school starts at age 7. Prior to that, children can attend a year in the optional preschool class. Preschool class serves to make the transition from preschool to school smooth since the traditions of play in preschool and the focus on learning in school otherwise can become problematic (Pramling & Pramling Samuelsson, 2008). Before 2016 there were no specific goals for preschool class in the curriculum

why the mathematics content and the design of the teaching differed a lot between preschool classes (National Agency for Education, 2014, 2016).

The study has been ongoing for four years and is conducted through educational design research including several design cycles with the stages of defining, testing and adjusting an intervention (McKenney & Reeves, 2012). The intervention consists of problem solving lessons that are implemented in several preschool classes. In this paper we start with results from the initial design cycle in which we found that children who used iconic representation when working on a combinatorics task made made more duplicate combinations than children using pictographic representations (Palmér & van Bommel, 2016). We will compare these result with results from a later design cycle when a special designed digital application was added to the intervention (van Bommel & Palmér, 2017). The initial design cycle involved 87 children from six preschool classes, the later design cycle involved 61 children from eight preschool classes. The children's guardians were given written information about the study and approved their children's participation in line with the ethical guidelines provided by the Swedish Research Council (2011).

The initial design cycle

When introducing the task, the children were shown three small plastic bears, one red, one yellow and one green. After the introduction the children worked individually. They were given white paper and pencils in different colours but no instructions regarding what or how to do any documentation on the paper. It is these documentations that we have analysed and Table 1 shows the categorization of this documentation.

	Pictographic	*Pictographic & Iconic*	*Iconic*	*Total*	
Some/All permutations – with duplications	3		30	33	33
Some permutations – no duplications	18 (4)	8 (3)	26 (10)	47	49
All permutations – no duplications			2	2	
Total	*21*	*8*	*58*	*87*	

Table 1 Categorization of children's documentation in the initial design cycle

There were more children using iconic representations than pictographic representations and 8 children used both pictographic and iconic representations. The documentations categorized as *Some permutations – no duplications* seldom contained more than four unique combinations. The numbers in brackets indicate those documentations where a child listed three unique permutations with each bear on each place only once (Figure 1, left). Only 2 of the 87 children found six unique permutations and they used iconic representation. However, 30 of the 58 children who used iconic representation made duplications (Figure 1, right) while only 3 of the 21 children who used pictographic representation made duplications. Thus, it seemed that the use of pictographic representations was more fruitful, in regard to solving the task, than the use of an iconic one. This

despite the fact that iconic representations most often are considered to be more advanced than pictographic representations (Hughes, 1986; Heddens 1989).

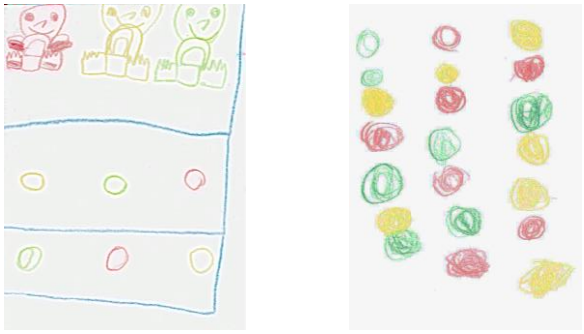

Figure 1: Two examples of children's documentation
Left: Pictographic & Iconic; Some permutations – no duplications (3 unique permutations)
Right: Iconic; Some permutations – with duplications

Moreover, according to English (2005), the major difficulty for young children, when solving combinatorial tasks, is listing items systematically. One possible explanation for the results could be that the main issue in solving this task is not about the representation used (pictographic and/or iconic representations) but about the systematization of the representations. Regardless of using pictographic or iconic representations, the children need to keep track of what combinations they have and have not drawn. Our further analysis indicated that there may be some connection between the representation the children use and their systematization (Palmér & van Bommel, submitted). At a first glimpse, it looked as if iconic representations did not generate a higher outcome of solutions of the combinatorial task; quite the opposite, pictographic representations resulted in a higher amount of unique permutations. However, the majority of children who showed systematizing in their documentations used iconic representations. Thus, the development of representations and systematizations seemed to be somehow synchronized however, an early use of iconic representations did not seem to support the development of systematizations. This possible, but not yet fully explored connection, led to the development of a digital application to be added to the intervention.

The digital application

To further investigate possible connections between representations and systematization we developed a digital version of the task. This digital application offers a semi-concrete pictographic representation (Hughes, 1986; Heddens, 1986) together with a systematic way of documenting each permutation (van Bommel & Palmér, 2017). The issue of duplications is included to the extend that if a previous documented permutation is selected again, the application indicates this with a red frame (see third image figure 2). The images in figure 2 show the semi-concrete representation within the digital application (an image of bears on a sofa), as well as the systematic way of documenting the permutations in the frames on the right hand side. In the first image, the child has only placed one bear on the sofa, in the second image, the child has completed one permutation which is visible in the little frame on the right hand side of the image. In the third image, the child

has accomplished three permutations and the fourth attempt resulted in a previously obtained permutation which is made visible through the red frame to the right.

Figure 2: Sequence of images of the digital application

The intention was to let the children work with the digital application before introducing the paper and pencil version of the task, and, through this, to investigate *if* and *how* the use of this digital application influenced the systematization and representation young children spontaneously use when they work on the paper and pencil version of the task.

Results - the later design cycle

In total, 61 children from eight preschool classes used the digital application before working with the paper and pencil version of the task. Table 2 below is a categorization of these children's paper and pencil documentation of the task, based on English's (1996) stages (trial-and-error, transition and odometer) as well as on Hughes' (1986) notions for representation (pictographic and iconic). When analysing systematization, we looked at the order of the permutations in each documentation. Could we for example see that one item had been kept invariant (transition), that one item had been varied (odometer) or did the permutations seemed to occur randomly (trial and error). Thus, the analysis of systematization is made from an observer perspective and it is possible that children had systematizations not visible to us.

	Pictographic	*Pictographic & Iconic*	*Iconic*	*Total*	
Trial and error – with duplications	1	1	3	5	*12*
Transition – with duplications		2	5	7	
Trial and error – some permutations – no duplications	2	1	2	7	*49*
Trial and error – all permutations – no duplications		1	1	2	
Transition – no duplications	5		8	13	
Odometer – some permutations – no duplications	9	1	6	16	
Odometer – all permutations – no duplications	3	1	9	13	
Total	*20*	*7*	*34*	*61*	

Table 2: Categorization of children's paper and pencil documentation

As can be seen, 20 of the 61 children used merely a pictographic representation and only 1 of these documentations included duplications. At a total very few children made duplications, only 12 of 61. Notable is also that 2 of the documentations categorized as *trial and error – some permutations – no duplications* as well as 6 documentations categorized as *transition – no duplications* included five unique permutations. The 16 documentations categorized as *odometer – some permutations* consisted of exactly three combinations, each bear sitting one time at each place. Two of the children using a trial and error approach and who made no duplications found all six permutations. Thus, at a total 15 of 61 children found six unique permutations.

Discussion – comparing the two design cycles

At this stage it is interesting to compare the results from the initial design cycle with the results from the later design cycle. In the initial design cycle 2 of 87 children found the six unique combinations, in the later design cycle 15 of 61 children found the six unique combinations (two on a trial and error level and 13 on an odometer level). In the initial design cycle 33 of 87 children made duplications (38%) while in the later design cycle only 13 of 61 children made duplications (19%). In the later design cycle 49 of the documentations showed a transition or odometer level indicating the application promoting systematization in the children's paper and pencil work. 2 of the documentations from the later design cycle categorized as *trial and error – some permutations – no duplications* as well as 6 documentations categorized as *transition – no duplications* included five unique permutations. Documentations with that many permutations without any duplications was unusual in the initial design cycle. In the initial design cycle 17 of the 47 documentations *some permutations – no duplications* consisted of exactly three combinations, each bear sitting one time at each place. Similar, 16 documentations categorized as *odometer – not all solutions* in the later design cycle. According to English (1996) this is common for young children since the repeated selection in systematic combinatory goes against the wording "different combinations". Especially young children often interpret "different" as different in all aspects. They do not think that keeping one item constant and change the others ends up as a "different combination". Instead, when each bear has been sitting one time at each place they think of the problem as solved.

Implications for further research

The digital application offers a semi-concrete pictographic representation together with a systematic way of documenting each permutation. Thus, the children who start with using the digital application start to work on the semi-concrete level and have possibility to explore systematization. Based on the analysed documentations we cannot claim that the digital application influences children's paper and pencil documentation but at the same time nothing in the results speak against that the use of the digital application influences the systematization and representation young children spontaneously use when they are working on a combinatorial task. One could imagine that the children would not find the paper and pencil part of the task interesting or challenging after using the digital application but there are no indications of this in the results. Even though the children who used the digital application found more permutations and made fewer duplications

compared to the groups who had not worked with the digital application, still few children “solved” the task.

The digital application was designed to offer a semi-concrete pictographic representation together with a systematic way of documenting each permutation which could create a different understanding of the combinatorial problem, which the results seem to indicate based on the few duplications made by the children. One thing that was interesting was that all but one of the children from two of the classes who had worked with the digital application used iconic representation in their paper and pencil documentations, and opposite, almost all of the children from a third class used pictographic representation. This diversity is something that we intend to explore further by interviewing children about their choice of representation, in close connection to working on the task.

Finally, we want to emphasize that we do not understand these preliminary results as a choice between paper and pencil *or* digital application but as the results indicate; paper, pencil *and* digital application. Based on this we consider it to be justifiable to proceed with a larger study.

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