Multimodality as an approach for studying arguments in mathematics teaching with younger students

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In this paper we describe findings on how mathematical arguments are created during whole class communications with younger students. What we emphazise in the paper is the analysis per se, and we give account for a multimodal approach, while drawing on social semiotics. This theory is used together with Toulmin's model of argumentation. The research question is "What functions do different communicational resources have for the construction of mathematical arguments in classroom interaction with younger students?". We also discuss implications for the teaching and learning of mathematics in terms of how a multimodal approach may constitute a bridge between mathematical arguments communicated informally to the similar arguments communicated formally, in accordance with the discipline of mathematics.

Introduction

In this paper we investigate what functions different communicational resources, such as speech, symbols and drawings, may have in the construction of mathematical arguments in classroom interaction with younger students. We adopted a multimodal approach (Van Leeuwen, 2005) together with Toulmin's model of argumentation. Drawing on the findings we discuss implications for mathematics education in terms of how a multimodal approach may constitute a bridge between mathematical arguments communicated informally to the similar communicated formally, as in the discipline of mathematics.

Multimodality in research in mathematics education

In mathematics education (ME) literature, it is common that spoken and written language, together with mathematical symbols, are taken as main and/or pre-given communicational resources in investigations aiming at understanding aspects of mathematics teaching and learning. This may be shown through displayed excerpts in articles, which are based on speech, while other communicational resources, such as pictures, are included when they are assumed to be needed for understanding what is taking place, while at other times not are being taken into account. One example is Krummheuer (2007), where the interest is directed towards mathematical arguments in the early years of school. He discusses different methodological strategies to conduct research on mathematical arguments as part of classroom communication. Excerpts show sequences where students are taking part of the construction of arguments. The dominating resource for communication is speech, but here and there pictures and hand gestures are also accounted for. This can be described in terms of speech being handled as the primary resource, with a readiness for inclusion of other resources as needed for understanding. In the article's methodological discussion there is no emphasis on functions of communicational resources.

Another way to attend to different communicational resources in ME research is an interest in students' thinking in relation to different semiotic resources (see e.g. Radford, 2014). Sometimes the interest is directed towards a specific communicational resource in relation to how people express mathematics. Often the studied resource is hand gestures (e.g. Morgan & Alshwaikh, 2012). Morgan and Alshwaik studied the interest leading to students' and teachers' choice of communicational resource in a situation where they were working with software that programmed turtles' movements in three dimensions (3D). During the analysis, the authors identified different kinds of hand gestures appearing in the communication. One such movement involved the teacher or student twisting the hand to the right or to the left. The findings indicated that there were differences in how teachers and students respectively made gestures, where the teachers made more difference between everyday gesturing and the specific gesturing that was "needed" for communication regarding 3D programming. The students mixed everyday and specific gestures to a higher extent, but also benefitted from the teachers' clarity in their gestures.

In other research, aspects of classroom communication have been studied where functions of a broad range of communicative resources in multimodal ensembles (e.g. Selander & Kress, 2010) are the focus. One early example of this in ME is O'Halloran (2000), who investigated three ways of expressing mathematics; mathematical symbols, visual figures of various kind, and language. The author describes the effect that the multi-semiotic character of mathematics has on classroom communication. In Boistrup (2015), the interest is on feedback in the mathematics classroom. The findings show how feedback is taking place with different communicative resources, sometimes without words, but with facial expressions and bodily movements. Furthermore, it is shown how students express mathematics with different communicative resources, and that they may be challenged mathematically by, occasionally, a limitation of resources to use. This may, for example, concern a student who displays knowledge about fractions through the use of manipulatives but is positively challenged in the learning when encouraged to use pictures, words and symbols.

This paper belongs to this last category of research through an interest in different communicative resources and the functions they may have for mathematical representation in the construction of mathematical arguments. The research project, from which the analysed excerpts derive, addresses mathematical arguments in classroom communication students grade 3-5, and was conducted according to a multimodal approach (Nordin, 2016; Nordin & Boistrup, submitted). In the project, three different categories of arguments were presented along with detailed descriptions of the situations analysed and with characteristics of the arguments.

Theoretical perspective

One theoretical perspective in this paper is social semiotics (Van Leeuwen, 2005). In this perspective, there are no communicational resources assumed to be the most meaningful in an interaction. In classroom communication, there are often many communicational resources that create multimodal ensembles (Selander & Kress, 2010), such as speech, writings, hand gestures, figures and symbols. When adopting this perspective for analysis, there is an openness for non-verbal communication, with for example only gestures. Focus is on the communication per se, and no claims are made of what a sign may "stand for" in terms of thinking or the like (Van Leeuwen, 2005).

Another theoretical perspective adopted in this paper is a model for argumentation developed by Toulmin (2003, see also Toulmin, Rieke & Janik, 1979). Toulmin's model of argumentation is frequently adopted in order to investigate arguments (as well as argumentation and reasoning) in educational settings. In addition, and particularly in mathematics education, many have followed the work of Krummheuer (1995) and his use of a reduced version of the model by Toulmin. The core of an argument, according to the model, consists of at least three elements: claim, data, and warrant (and potentially a fourth element, backing). The claim is a statement that can be grounded by data and warrant, where the warrant functions as a bridge between the data and the claim/conclusion (figure 1).

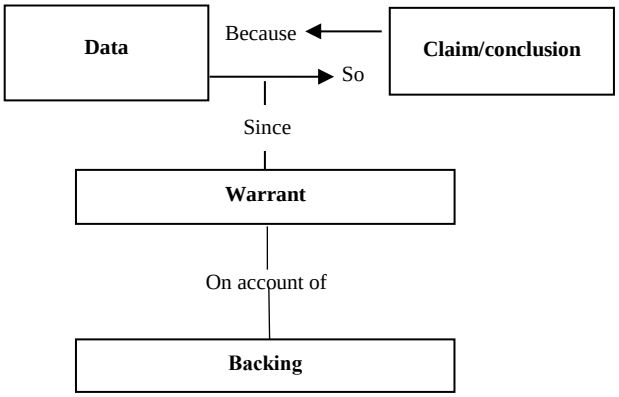


Figure 1: A model of argumentation (Toulmin, 2003)

In order for statements to be regarded as part of an argument, someone makes some kind of *claim*. This could, for instance, be an answer to a problem. It is also required that the people in the interaction display that they have built this claim on "something", on some kind of *data*. Moreover, in order for it to be regarded as an argument, the link between the data and the claim must be clarified, through a *warrant* of how the claim can be made on the basis of this data. It may also occur that the motivation, the warrant, is backed up in a *backing*. This means that there is an extra clarification on why this motivation, this warrant, is applicable. The model is further explained in relation to excerpts 1-5 below. The statements are not needed to occur in the same order as in the model, as long as the different elements of the model can be reconstructed in the analytical process.

Methods

In this study we have analysed excerpts from a previous publication by us (Nordin & Boistrup, submitted), mainly deriving from video recordings of mathematics teaching in grade 3-5, during whole class sessions. The interactions in the videos were transcribed multimodally, and were analysed with an interest in types of arguments. In this paper we have performed a reanalysis of the excerpts from this submitted publication, but with an interest in functions of communicational resources.

We analysed the multimodally written excerpts (see excerpts 1-5) looking for affordances of each communicative resource. We marked the resources that constituted each element of the reconstructed arguments, the claim, data, warrant, and backing. We then interpreted whether respective resource, in the specific situation, was a primary resource conveying a mathematical argument, if it was used equally with another resource, or if was a secondary resource (explained below).

Analysis and findings: first round

In the first round of the analysis we drew on multimodal social semiotics (Van Leeuwen, 2005) and construed the following categories for discerning which emphasis a communicative resource had in a statement: *Primary* (P): The resource conveyed the meaning very much on its own. Typically it was the first in terms of order in a specific statement; *Equal* (E): The resource equally, and simultaneously, conveyed the meaning together with one or more other resources; and *Secondary* (S): The resource helped in conveying the meaning, while not being an essential resource in the particular statement.

The communicative resources, analysed in terms of their functions in the construction of arguments, where the following: *Speech* (words), *Figure/drawing* (included both people making figures and drawings, and people showing these), *Symbols* (numbers, equal signs and the like), and *Hand gestures* (pointing with finger or with hand towards, for example, a figure).

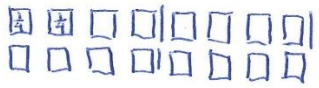
It is quite common that elements, such as claim and data, come in a different order in real world arguments compared to how the model by Toulmin (2003) is described above (figure 1). In this paper we instead present the argument in the same order as we have described the Toulmin model, starting with the *claim*, then giving account for the *data*, followed by the *warrant*, and finally the *backing*. For each such element we describe the role of the communicative resources, whether they had a *primary* function in conveying the meaning or if they were *equal* with other resources, or *secondary*.

Analysis and findings: second round

For this second round we give account for one argument in detail. The argument was reconstructed in Nordin and Boistrup (submitted, see also Nordin, 2016) from a lesson in grade 4 where this problem was discussed:

> It is a field day and it is sunny and warm. The school will provide food and drinks. Each student is given a quarter of a litre of juice to drink. There are 16 students. How much juice will be needed?

Previous groups had described their solution of the problem when it was Frida and her friend's turn to do the same. In the situation both students are standing in front of the class. As described above, we do not follow the chronological order of the argument in our account as it was made during the communication. Excerpt 1 is, for example, taking place in the end of the argument being made, when Frida showed her *claim* of the argument.



Excerpt 1. Frida displays the answer to the task

The *claim* (in this case the answer to the problem) in this argument was interpreted as conveyed by a *drawing* made by Frida (excerpt 1). In the drawing it is shown that four litres of juice were needed for the field day. The 16 mugs needed for the students were divided into four groups, which correspond to the four litres needed. Since only one communicative resource was used, *drawings*, it was analysed to be the *primary* communicative resource for conveying the *claim*. As will be shown below, the drawing in excerpt 1, or parts of it, was interpreted to communicate, along with other communicative resources, also other elements of the argument.

Data was interpreted from excerpt 2, which is from the very beginning of Frida's presentation.

	Speech	**Writing/drawing/object**	**Hand gestures**
24:32 24:39	Frida (F): I started to draw all students' mugs.	F draws a rectangle/mug	F points at the rectangle/mug with a pen.

Excerpt 2. Frida starts drawing mugs

In excerpt 2 it is shown that Frida started drawing the mugs. We can also see that she pointed at the drawn "mug", when she spoke. In this respect the drawing of the 16 mugs constituted *data*, representing the 16 students, which she drew on for creating her argument. The act of *drawing* was analysed to be the *primary* resource here and *speech* and *hand gestures* as *secondary*. In excerpt 3 another *data* is presented.

24:42	T: How much was it in each mug then? F: Er…	F draws a second mug F writes ¼ in the first mug		
24:49-24:56	T: It was a quarter in each mug.	F writes ¼ in the second mug	T takes one of the quarters out of a magnetic circle and places it over the first mug drawn by the student.	

Excerpt 3. Frida and teachers states that it is ¼ in each mug

In excerpt 3 Frida and the teacher made the *data* clear that there will be ¼ juice in each mug. Frida's conveyed this *data* when writing ¼ in two mugs. *Symbols* were here interpreted to be the *primary* resource. The teacher's part of the *data* was when she *spoke* at 24:49, while showing through a quarter of a magnetic circle (*figure*). *Speech* and *figure* were considered to be *equal* resources used here. From excerpt 4, the *warrant*, the link between *data* and *claim* was interpreted.

25:28	F: I did like this.	F draws a line after four mugs after eight mugs as well as one after four mugs on the second row. 	

Excerpt 4. Frida motivates her answer to the problem

What is analysed in terms of *warrant* is how Frida drew the lines described in excerpt 4, and this way showed how the *data* (16 mugs and ¼ in each mug) provided a basis for the *claim* that four litres were needed (analysed in excerpt 1). The *drawing* of the lines were here interpreted as a *primary* resource, while *speech* was a *secondary* resource ("I did like this").

From excerpt 5, the *backing* was analysed, which was stated by the teacher, supporting the *warrant*.

27:23	T: We do know that in (one) litre there are four quarters	T puts the four quarters of the magnetic circle to a circle	

Excerpt 5. The teacher explains, using a magnetic circle

As shown in excerpt 5, the teacher used *speech* to *back* up the *warrant*, saying that in one litre there are four quarters. This way she explained why the *warrant* that Frida made was valid. Simultaneously, and *equally* significantly, she showed how the four magnetic quarters made a whole circle (*figure*).

Analysis and findings: third round

In table 1 we present a summary of the argument described above. The number of statements are summarized in relation to which communicative resource was conveying specific elements of the argument.

Table 1: Frida and the mugs (number of statements)

	Speech	**Figure/drawing**	**Symbols**	**Hand gestures**
Claim		P: 1 **T: 1**		
Data	E: 1, S: 1 **T:2**	P: 1, E: 1 **T: 2**	P: 2, **T: 2**	S: 1 **T: 1**
Warrant	S: 1, **T:1**	P: 1, **T: 1**		
Backing	E: 1 **T:1**	E: 1 **T: 1**		

* P=primary resource, E=equal resource with other(s), S=secondary resource, T=total

From table 1 we read that in the situation with Frida and the mugs, a broad range of communicative resources conveyed the argument. Figure/drawing was the resource that conveyed an element very much on its own (primary resource).

In table 2-4 we show the summaries of the three other arguments described in Nordin and Boistrup (submitted). *Backing* was not interpreted in all arguments.

Table 2: Alex figures out the price of one ice-cream

	Speech	**Figure/drawing**	**Symbols**	**Hand gestures**
Claim	P: 1, **T:1**			
Data	P: 1 **T:1**	S: 1 **T: 1**	S: 1 **T: 1**	S: 1 **T: 1**
Warrant	P: 1 **T:1**	S: 1 **T: 1**	S: 1 **T: 1**	S: 1 **T: 1**

Table 3: Teacher posed questions about fractions

	Speech	**Figure/drawing**	**Symbols**	**Hand gestures**
Claim	P: 1, **T:1**			
Data	S: 1 **T:1**	P: 1, E: 1 **T: 2**		
Warrant		P: 1 **T: 1**		

Table 4: Kim showed his calculations

	Speech	**Figure/drawing**	**Symbols**	**Hand gestures**
Claim	E: 1 **T: 1**		P: 1, E: 1 **T: 2**	E: 1 **T: 1**
Data	P: 2, E: 2 **T: 4**		P: 3, E: 2. S: 1 **T: 6**	E: 1, S: 2 **T: 3**
Warrant			P: 1 **T: 1**	
Backing	P: 1 **T: 1**		S: 1 **T: 1**	S: 1 **T: 1**

When analyzing the four tables, it is clear that they all are different in terms of what functions different communicative resources had in the construction of arguments. *Speech* was, not surprisingly, present in all four arguments. In three arguments (tables 2-4) this resource was regarded as the primary one, at least partly. *Figure*/*drawing* was present in all but the last argument, with a primary function in two (tables 1, 2). *Symbols* was also present in three arguments (table 1, 2, 4) and rather dominating in the argument where a student showed his calculations (table 4). Finally, *Hand gestures* were present in three arguments (table 1, 2, 4) conveying a statement equally with other resources or as a secondary resource.

A conclusion we made is that a broad range of communicative resources may play the function of conveying parts of arguments, as they are created during classroom communication, specifically when the students are young. We also conclude that the multimodal ensemble (Selander & Kress, 2010), conveying elements of arguments, is likely to be very different between different kinds of situations. In a situation where the students solve problems and they present their solutions to the class with help from the teacher, reflected in table 1, figures/drawings may have the function to primarily convey the argument, supported by speech, symbols and hand gestures. For another student and/or problem, a primary resource may be speech, supported by other resources. This was the case for Alex who explained the solution to a problem about prices of ice-creams (table 2). Another argument may concern the explanation of calculations. Here symbols may get the primary function in conveying the argument. This was the case for Kim (table 4). He first explained his argument only by writing symbols on the white board. Subsequently, he combined speech while pointing at the calculations written earlier. As a contrast to Frida, figure/drawing was not used at all in this argument. Finally, the teacher, in table 3, but also in the other situations, only used speech and figure/drawing to convey arguments in the excerpts analysed for this paper.

Discussion

While there is much literature where a multimodal approach is adopted, not much focus has been on functions of communicative resources in the construction of mathematical arguments Through analysed excerpts and analyses summarized in tables we have illuminated how a multimodal approach is essential if we, as researchers and/or teachers, really want to capture and acknowledge mathematical arguments created in day-to-day mathematics classroom communication. Some of the arguments analysed by Nordin and Boistrup (submitted) would likely not have been regarded as arguments at all with a more traditional approach of classroom research, relying on mainly on speech

and symbols, with the addition of other resources when "called for". Frida in excerpt 1-5, is an example of an argument that would have run the risk to be missed. She did not speak much, and the main resource for conveying elements of the argument was figure/drawing.

An implication is, however, that even though it is essential to capture arguments conveyed through a broad range of communicative resources, the teacher has a significant role in letting the students experience mathematical arguments described in symbols, which is contradictory to what is displayed in table 3. The multimodal approach illuminated in this paper may serve as a tool in order to discuss mathematical arguments conveyed firstly by, for example, drawings and then, in parallel, in symbols. This approach may then constitute a bridge between mathematical arguments communicated informally to the similar arguments communicated formally, as in the discipline of mathematics.

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