Collaborative tool-mediated talk – an example from third graders

Heidi Dahl, Torunn Klemp and Vivi Nilssen

Norwegian University of Science and Technology, Norway;

heidi.dahl@ntnu.no, torunn.klemp@ntnu.no, vivi.l.nilssen@ntnu.no

In this paper, we investigate a dialog between two eight-year old girls solving a mathematical task. The study is video-based, and carried out within a sociocultural framework. The analysis shows that the girls' communication skills and use of tools has a profound impact on their potential to solve the task as a joint enterprise.

Keywords: Mathematical problem solving, use of tools, representations, collaborative talk.

Introduction

This paper is based on classroom observations in Norway where pupils in third grade are working on a multiplication task. The study is part of a larger research and development project called *Language Use and Development in the Mathematics classroom* (LaUDiM). The main objective of the project is to develop deeper knowledge of the learning environment's significance for developing young learners' mathematical thinking and understanding, as well as to develop their ability to express mathematical concepts and ideas. Amongst the research questions, one is to understand more about how young pupils collaborate on solving mathematical tasks.

Theoretically (Vygotsky, 1987) and research-based (Mercer & Sams, 2006), the importance of language and social interaction in learning mathematics has been emphasized. This is also a claim in the Norwegian national curriculum for primary school (LK06). There are, however, some precautions from researchers arguing that just putting pupils together will not always work. The talk is often uncooperative, off-task, inequitable and ultimately unproductive (Mercer & Sams, 2006). Sfard and Kieran (2001) concluded that "interaction with others, with the numerous demand on one's attention, can often be counterproductive. Indeed, it is very difficult to keep a well-focused conversation going when also trying to solve problems and be creative about them" (p. 70). They argue that strong motivation is necessary to engage in mathematical conversations and make it work, and a prerequisite for a mathematical discourse to be productive is the effectiveness of the communication among partners. Van Oers (2013) claims that there is a need to find out more about what productive dialogs that support mathematical thinking and learning entail.

In this paper we present, analyse and discuss a dialog between two eight-year old girls. The dialog ended with the exclamation "Yes, we did it" which we took as a preliminary evidence of a successful collaborative talk. Thus, the research question for this paper is: What stimulates mathematical progress in the collaborative process of solving a task?

Theoretical framework

Our point of departure is sociocultural theory as developed by Vygotsky (1987) and his successors. Two important features of this theory are particularly relevant for our study. First, the claim that higher mental functioning, like reasoning and problem solving in the individual derives from social life. Second, that higher mental functioning and human actions in general are mediated by tools and signs. Vygotsky's accounts of mediation provide the bridge that connects the external with the internal and thus the social with the individual (Wertsch & Stone, 1985). Vygotsky viewed language to be the most important tool, both for the development and sharing of knowledge among people and also for structuring the process and content of individual thought. From a sociocultural perspective, it is particularly interesting to study talk in educational settings and identify in what ways humans learn to handle and use cultural tools effectively to solve problems.

Exploratory talk is a typification of a way of using language effectively for joint, explicit, collaborative reasoning (Barnes & Todd, 1977, Littleton & Mercer, 2010). In exploratory talk knowledge is made publicly accountable and reasoning is visible. It represents a form of co-reasoning where speakers share knowledge, challenge ideas, evaluate evidence and consider options in a reasoned way. Explanations are compared and joint decisions reached. "It is a speech situation in which everyone is free to express their views and in which the most reasonable views gain acceptance." (Littleton & Mercer, 2010, p. 279). According to Barnes and Todd (1977) exploratory talk depends on learners who share the same idea of what is relevant to the discussion and have a joint conception of what is trying to be achieved by it. Two other kinds of talk are presented by Littleton and Mercer (2010). In *cumulative talk*, speakers build positively but uncritically on what the others have said. It is characterized by shared information, joint decisions, repetitions, confirmations and elaborations, but there are no critical considerations of ideas. *Disputational talk* is characterized by disagreement and individualized decision making with few attempts to combine resources, offer constructive criticism or make suggestions. There can be exchanges of these three types of talk.

Duval (2006) claims that all mathematical activity involves the use and change of semiotic representations. He introduces a classification of semiotic representation into four different registers; natural language, symbolic systems, iconic and non-iconic drawings, and diagram and graphs, based on the possibilities for performing mathematical processes. Natural language has a special position amongst the registers, as it can be used also for communication, awareness, imagination etc. Duval denotes transformations between representations within the same system as treatments, and transformations between different registers as conversions. He claims that conversions are more complex than treatments, "because any change of register first requires recognition of the same represented object between two representations whose content have very often nothing in common" (p. 112). Hence the ability to perform successful conversions is often a critical threshold for progress in problem solving.

The dialog presented in this paper is taken from a teaching sequence where the mathematical aim was to give the pupils experiences with different multiplicative situations. Steffe (1994) characterizes a multiplicative situation as one where "it is necessary to at least coordinate two composite units in such a way that one of the composite units is distributed over the elements of the other composite unit" (p. 19). Depending of the situation, four different multiplicative structures can be distinguished; equal groups, multiplicative comparison, rectangular area, and Cartesian product (Greer, 1992). The task involved in this paper concerns the first structure. In an equal group situation, the multiplicator counts how many groups are involved, while the multiplicand tells the number of objects in each group. Such situations are asymmetric problem situations, meaning that the role of the factors cannot be interchanged without reinterpreting the situation.

Methodology

LaUDiM is an intervention project where two teachers from different schools and researchers from the field of mathematics education and pedagogy plan and set goals for the teaching of mathematics, which subsequently is carried out by the teachers. In the classroom, whole class discussions and dialogs between selected groups of pupils are video recorded. Parts of these video recordings, together with pupils' written work, are discussed by researchers and teachers. This represents the first step in analysing data as interesting sequences are identified. The presented dialog is chosen from video-recordings of six collaborating pairs working on the same task. By carefully viewing all the recordings we chose this dialog due to the task-focused content, and to the engagement and passion we could see between the two girls. Moreover, the session ended as already told with the exclamation "Yes, we did it" which we took as a preliminary evidence of a successful collaborative talk.

The empirical data for this paper is a video-recorded and transcribed 7 minutes' dialog between the two girls, working on the task:

The 3rd grade will have a party at school. The day before the party, they are baking muffins for the party. Line is going to the store to buy eggs for the muffins. In the recipe, it says that they need four eggs in one portion. The children have decided that they are going to bake twelve portions of muffins. How many eggs does Line need to buy?

The girls' discussion is a collaborative effort to solve the mathematical problem. According to Blum and Niss (1991), a mathematical problem is a situation that challenges somebody intellectually who is not in immediate possession of direct procedures sufficient to answer the question.

To find an answer to the research question, we started by doing a conversational analysis. Keywords from Littleton and Mercer's (2010) characteristics of different types of talk served as guidelines in this process. Examples of questions asked to the material are how do the girls respond to each other, how do they give reason and how do they share ideas. Due to the video-based design of the study, we were able to identify not only their oral talk, but also use of gestures and other mediational tools. The next step was to identify shifts of focus in the dialog. This helped us to divide the dialog into sequences, which were analysed further with respect to the mathematical content. In this process, use and shifts of representations became visible. This turned our attention to Duval's (2006) work on this issue. In the third step, we analysed and interpreted each sequence more thoroughly by combining the two analytical perspectives. We have decided to present and analyse the dialog as it unfolds, just leaving out a few utterances we find unnecessary.

Analysis of the dialog

The dialog starts by Tea reading the word-problem aloud, until Lin interrupts her.

1	L:	I'll draw four eggs?
2	T:	Wait, wait (continues to read the task aloud).

(…)

7 L: I'll just draw some circles (starts to draw a row of small circles).

8 T: Draw four circles. There you are. Good. And then we should..., and then we have twelve…, just write twelve, no, forget it.

While Tea is still reading the word problem, Lin suggests a conversion from the problem stated in natural language to an iconic representation (1, 7). Tea supports this transformation, by monitoring and evaluating Lin's action (8). The girls are at this point unsure of the role of the number 12, and it is not likely that they recognize the problem as multiplication at this point. Tea then goes back to the written task, and after some thinking time, the conversation continues.

13 L: This is an addition problem.

14 T: No, (whispers) it is 12 times 4.

15 L: Oh, yes.

16 T: No, it's 4 times 12

17 L: (Laughs) Yes, that is the same.

18 T: It is 4 times 12, …no, it is not the same. For if we take 12 times 4, then we take 12 four times[1].

19 L: Yes.

20 T: And that does not work here.

At this point it seems as if the girls have given up pursuing the iconic representation, instead they try to find a number sentence that fits the word problem, indicating a conversion from natural language to mathematical symbols. As Lin is not given the chance to explain her thinking (13), it is not clear whether she makes a successful conversion, suggesting repeated addition of 4's. Eagerness to explain the difference between 12·4 and 4·12 (18), is taken as an account for that it is important to Tea that Lin follows her reasoning. Recognizing the situation as multiplicative gives Tea some new input on how the problem situation can be modelled, and so the problem-solving moves on.

22 T: (Points at the four eggs) So that means four…, we should get to… we are going to have twelve. (Takes the paper from Lin.) If I draw twelve.

(…)

25 L: Just do it there (points right beneath the four eggs).

26 T: I'll draw twelve muffins[2] (starts to draw bigger circles, stops to count).

27 L: That's funny looking muffins.

28 T: I know, but we can see, we can see what it is anyway (completes the drawing of twelve muffins; two rows with six circles in each row).

29 L: Now you have twelve.

30 T: Here we have twelve muffins, and then there should be four in each muffin (points at the eggs Lin has drawn at the top of the paper).

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[1] Tea is aware of the difference between 4·12 and 12·4, but her interpretation does not follow the usual convention.

[2] There is some confusion between muffins and portions, but that is not important for the solution.

31 L: (Points at the four eggs) Then we put these down here, these four in one, then we have to… (points from the four eggs to the twelve muffins).

Tea identifies that the muffins are the essential units to start with in the iconic representation, and she makes the crucial connection between the muffins and the eggs by pointing at Lin's drawing of four circles (30). This shows that she has grasped the multiplicative structure of the problem, one unit distributed over the other, and is thus a mathematical breakthrough. The gesture also serves as an acknowledgement of Lin's contribution. Lin actively monitors Tea as she draws the muffins (29), and by suggesting to "put down" the eggs (31) she lets Lin know that she both understands the structure of the problem and approves of her representation of it, and the girls are ready to proceed.

34 T: Because in this, if we add them together we get eight. (Points to the first muffin in each row, writes the number 8). Because in each there is eight.

35 L: Here, just read from here again. Slowly.

(…)

39 L: Stop. We need four eggs in a portion, right?

40 T: Yes, because one portion, that is one muffin for us then (points at herself). So that means that in this one there is four (points at the first of the muffins).

41 L: (Points at the four eggs) all of these circles here, just draw a line down to… (Points at the first of the muffins).

42 T: In one there are four, and in that one there are four, so if we add them, we get eight.

43 L: I'll take four of them in here (draws four small circles inside the first muffin).

44 T: No, just... I will… (takes the pencil from Lin). Eight plus four, we do it like this, four, four, four (writes the number 4 above each muffin).

45 L: Can I do the last ones?

46 T: Yes, you can do these four.

47 L: Oh no (draws a negligent looking 4).

48 T: That's fine, that's fine, we can see it anyway.

Having seen through the multiplicative structure of the task, Tea seems ready to use the representation of the twelve muffins to start calculating, keeping the number of eggs in each muffin as a mental image (34). Lin on the other hand, needs a more concrete representation of the eggs (41, 43). They compromise by writing "4" over each muffin (44). After confirming that they share a common understanding of the new representation, they continue.

52 T: No, look here, do you know what, wait, we have to do it again now, because…, if we take… (Points to and counts the six muffins in the first row) this is six, right (writes 4+4+4+ on a line below the drawing of the muffins). Now I have taken these three (puts a mark after the first three muffins, counts as she writes more +4's) 1, 2, 3, 4, 5, 6, 7, 8.

53 L: (Counts the muffins silently.) Just take twelve of those. Ok, I'll just read (takes the problem sheet, reads to herself, following the text with her finger).

54	T:	(Counts aloud, finishes to write +4) 9, 10, 11, 12. Ok, here I made a plus-problem with all these (points to the muffins). Then we have twelve fours, just that…, here we have the answer (writes =___ below the row of +4's).

Both girls are able to use the drawing of the muffins, combined with the rows of 4's, to start a process of repeated addition, but they face some challenges keeping track of the preliminary calculations. Tea takes the lead of transforming into a more structured symbolic representation (52), thinking aloud to ensure that the group agrees. Lin is not passive in this process, she monitors Tea's work, and checks once again that the representation they have come up with is in line with the written task (53). After some negotiation on the notation, the girls are ready to perform the needed calculations.

63	L:	It is 16 (points).
(…)		
66	T:	Ok, ok I believe you. Plus four, 16… (Draws more vertical lines and writes 16), and here we have four.
67	L:	16
68	T/L:	(Both counting on their fingers) 17-18-19-20 (Tea writes 20).
69	L:	24 (Tea writes 24), 28 (Tea writes 28)
70	T:	(Counting on her fingers) 29-30-31-32 (writes 32)
71	L:	(Counting on her fingers) 33-34-35-36 (Tea writes 36)
72	T/L:	(Counting on their fingers) 37-38-39-40 (Tea writes 40), 41-42-43-44 (Tea writes 44)
73	T:	Oh, that one, that one we could have done right away.
74	L:	48 … I think.
75	T:	Yes, it is 48.
76	L:	Yes, it is 48. (Tea writes 48 behind =). So, we have to buy 48. Yes, we did it!

The new representation works for calculating and the girls share the same strategy, taking turns counting in fours. They use their fingers as support, but the counting is rhythmic, so they might be capable of using an internal count. When there are only a few more fours to add, they turn into a choral count, which indicates that they are enthusiastic as they approach an answer. Lin's "Yes, we did it" shows pride of having fulfilled their common project.

Discussion

What stimulates mathematical progress in the collaborative process of solving the task? To be able to answer this, we first identify what comprises the mathematical progress in the dialog. We then show how combining our two analytical perspectives contributes to answer the research question.

The solution process is not straight forward for the girls. Anghileri (1989) claims that multiplication differs significantly from addition in complexity because there are three pieces of information to coordinate; the number of sets; the number of elements in each set; and the procedure for executing the product. The mathematical progress in the dialog can be described in two steps. First, the mathematical breakthrough happens when the girls identify the multiplicative structure of the problem situation (30, 40-43). They recognize that the group of eggs constitute a composite unit that is to be distributed over the muffins. The task can then be solved by repeated addition of 4's. The girls' actual calculation constitutes the second step of the mathematical progress. This, of course, leads them to the final answer, but identification of the multiplicative structure is crucial in order to be able to start the calculation. The analysis shows that when the girls are stuck in the process of solving the task, they use two strategies to make progress; they either re-read the task, or they perform a shift of representation (Duval, 2006). By constantly going back to the written problem the girls check that they have a joint conception of what is trying to be achieved (Barnes & Todd, 1977), while the change of representations serves as a tool that helps them uncover the structure of the task, to perform calculations, and to structure and communicate their thoughts. The girls' need of a model of the problem situation as a tool for thinking is in line with previous research on young children's pre-instructional multiplicative strategies (Kouba, 1989).

First and foremost, the mathematical progress in the dialog is stimulated by the fact that the girls have a common goal in solving the task (Sfard & Kieran, 2001). The repeated use of "we" instead of "I" indicates that they share the responsibility for the project. There is an atmosphere of trust and acknowledgement between them, visible for instance when Tea gives positive feedback on Lin's drawing (8), when they don't mind that their drawings are not perfect (28, 48), and when Tea trusts Lin's calculation (66). This mutual acceptance is a necessary condition for co-reasoning, as it creates a space where the girls dare to share ideas.

Two characteristics of the girls' communication seem especially important for stimulating mathematical progress; the girls' ability to think aloud, and their eagerness to actively involve themselves in each other's reasoning. First, making their thinking public makes it possible to follow each other's reasoning, to evaluate it, and build upon it. An example is when Lin draws four eggs, stating aloud what she is drawing. Tea then tries to build upon Lin's work, but is unsure of the role of the number 12 (1-8). Thinking aloud also enables the one sharing her idea to think it through more thoroughly, leading to a deeper insight (Vygotsky, 1987). An example of this is when Tea explains the difference between $4 \cdot 12$ and $12 \cdot 4$ (18). Almost immediately it seems like she sees the connection between the pair of numbers and an iconic representation of the problem, making her able to model the situation in a way that illustrates the multiplicative structure. The thinking aloud in this dialog seems to be especially interrelated with the use of drawings and other written representations. It is striking that whenever a change of representation is performed, the girls very carefully explain their actions. We see this when Tea makes the drawing of twelve muffins (22-30), and later when she turns the problem into a repeated addition problem (52-54). Making their thinking public in these situations is especially important because the written representations are the dominant mediational means in the solution process. On one hand, one can say that the mathematical progress is dependant of the girls' ability to accompany their written work with verbal explanations, as this may contribute to a shared understanding, crucial for keeping the solution process a common project. On the other hand, the drawings act as means to elicit the girls' thinking aloud, giving their verbal reasoning a necessary support. In a way, the drawings and the girls' ability to think aloud seems to be interdependent.

Secondly, the girls constantly involve themselves in each other's reasoning, either by monitoring each other's actions, as when Lin confirms that Tea has drawn exactly 12 muffins (29), or by actively participating in the other's construction of a new representation (45). This involvement is important for mathematical progress because it ensures that the reasoning is supported and understood by both participants, and hence serves as a green light to continue.

In "ideal" exploratory talk, ideas are often challenged or questioned. This does not happen often – if at all – in the dialog between Tea and Lin. As shown, this does not mean that they passively accept each other's ideas. As the two girls are using language effectively for joint, explicit, collaborative reasoning we claim that the conversation inherits features of exploratory talk (Littleton & Mercer, 2010). Our study adds to the field throwing light on how drawings are necessary mediational means in young learner's exploratory talk. The girls are involved in what we will call a collaborative tool-mediated talk in order to solve the mathematical task.

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